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Report on the Application of Finite Element Analysis to Historic Structures

Westminster Hall, London

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Monumental architecture and the design conceptions on which it is based inevitably have structure at their very core. The spacing of columns, dimensions of beams, thickness of walls, and placement of buttresses—the elements which define our spatial experience—all require consideration of structural parameters. As a result, the form and the structure of many great monumental works of architecture are tightly interwoven, so integrated that any attempt to study form or style in such buildings must be based on an understanding of the structural principles at work.

Modern historians exploring earlier building technology have several options regarding structure, its relationship to aesthetic developments, and its place in architectural history. The first is simply to ignore it; a common enough choice with the development of art and architectural history as a separate discipline as part of the liberal arts curriculum. For many historians, however, this is not an entirely satisfactory solution. Historic builders and designers striving to ensure the integrity of their constructions had no such luxury; their works inevitably grappled with both structural and aesthetic issues at once.

Alternatively, the researcher may choose to engage this relationship seriously, relying on his or her knowledge of the period's construction practices and techniques to interpret the significance of structural decisions in historic buildings. This, perhaps, has been the most common approach. For example, inquiry into the logic of a given structural configuration or device may provide a rudimentary understanding of a building's structural performance. With simple structures—and in this category fall most medieval timber roofs—this practice may be enough to highlight the relationships between design decisions and structural considerations. However, the accurate determination of a more complex building's structural behavior typically requires the technical knowledge of those trained in engineering. Hence, in the field of architectural history, this path has been pursued by only a few.[1]

Historians may also collaborate with engineers. Unfortunately, the engineering process (along with its specialized terminology and numeric answers to technical questions) has often discouraged nonengineers from active participation in analysis. Moreover, prior to the development of computer-aided analysis in the 1960s, engineering had some serious limitations for historical research: Lengthy hand calculations characterized the process, and to facilitate computation theoretical structural models were often developed on the basis of oversimplified assumptions.[2] Finally, as alternative hypotheses could not quickly be explored for their validity, they were not generally explored at all.

This paper describes the potential of another approach to structural insight, one in which computer–aided finite element analysis (FEA) is used to inform historical research directly. FEA is an interactive medium effective in exploring and discussing structural hypotheses. With minimal training, it is accessible to nonengineers. Its products include output useful to developing an understanding of a structure's overall behavior and details concerning the performance of its components. Most important, by enabling researchers trained in architectural history to participate directly in the structural analysis of historic buildings, it can highlight the relationships between formal or aesthetic issues and structural developments.

WESTMINSTER HALL

As an illustration of this method the authors used FEA to explore the trusses at Westminster Hall. Hugh Herland's celebrated roof (1395–96) is best known for uniting two hitherto separate roof prototypes—the hammerbeam and the arch-brace-and-collar roof—to create a structure spanning one-and-a-half times the distance of any previous timber roof [*Figure 1*]. It is also famous for its highly decorative traceried woodwork. How does the truss carry its loads and with what intent were its ornamental features employed? This structure of innovative design and outstanding beauty has been the subject of a great deal of speculation by historians, engineers, and architects alike.[3]

A century of numerous studies, analyses, and debate, however, has only begun to establish the overall load–bearing behavior of Herland's complex design. Beginning in 1914 with an exhaustive study by Sir Frank Baines (Baines's *Report*)

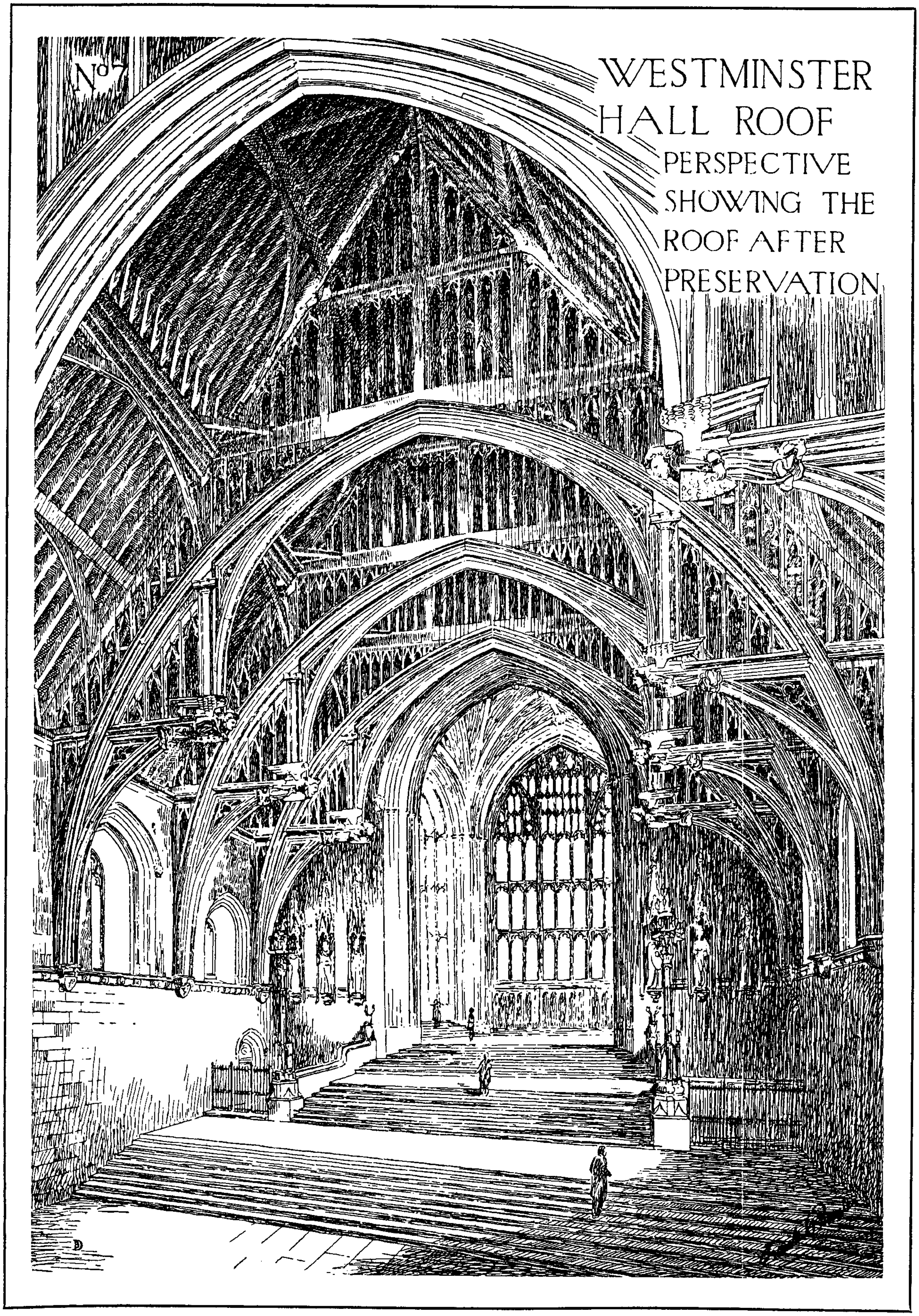


FIGURE 1: Sir Frank Baines, interior perspective, Westminster Hall roof, from *Report on . . . Westminster Hall* (1914).

documenting the condition of the roof trusses, twentieth-century investigators have attempted to explain the performance of the truss.[4] As architect in charge of roof restorations for Westminster Hall (1913–22) Baines was primarily interested in establishing the viability of the much decayed trusses and determining the most effective means of reinforcing them. His analysis of the structure was based primarily on his general knowledge of hammerbeam and timber–frame construction.[5] In 1926, A. J. Sutton Pippard and W. H. Glanville used the Westminster roof as a case study to demonstrate Pippard's pioneering "strain energy stress analysis" technique.[6] In 1967, engineers Jacques Heyman and Rowland Mainstone each took up the discussion and questioned which of the principal members in Herland's roof were structural and what constituted appropriate assumptions for analysis.[7] Most recently, in 1987 Robert Mark and his associates, in collaboration with Lynn Courtenay, conducted the first structural study of the trusses to combine both scale model and computer analyses.[8] Although all of these researchers have brought the best of available theory and science to the task of explaining Herland's remarkable technical achievement, there is as yet no definitive analysis.[9]

This study continues this tradition of structural investigation by bringing state-of-the-art engineering analysis to the problem of understanding the trusses' structural behavior. It differs from those preceding it, however, in one important respect: it includes the ornamental tracery in the analysis of possible structural components [*Figure 2*]. Either because of the difficulty of analyzing it with earlier techniques, or because of a bias held by engineers and historians which has viewed the tracery as purely ornamental, no one has studied the truss in its entirety.[10] This is understandable considering the tracery's slight section in comparison to those of the massive principal timbers which characterize the truss. The great arch, for example, composed of three pegged sections of English oak, is nearly two foot square in section, lending a powerful presence to the truss. By contrast, the tracery seen 60 and 80 feet above the floor presents a delicate filigree of detail tying together such principal timbers. Each individual puncheon of the tracery, however, is a sizable timber in its own right, and at roughly 4 by 9 inches in section is capable of sustaining thousands of pounds of compressive force.

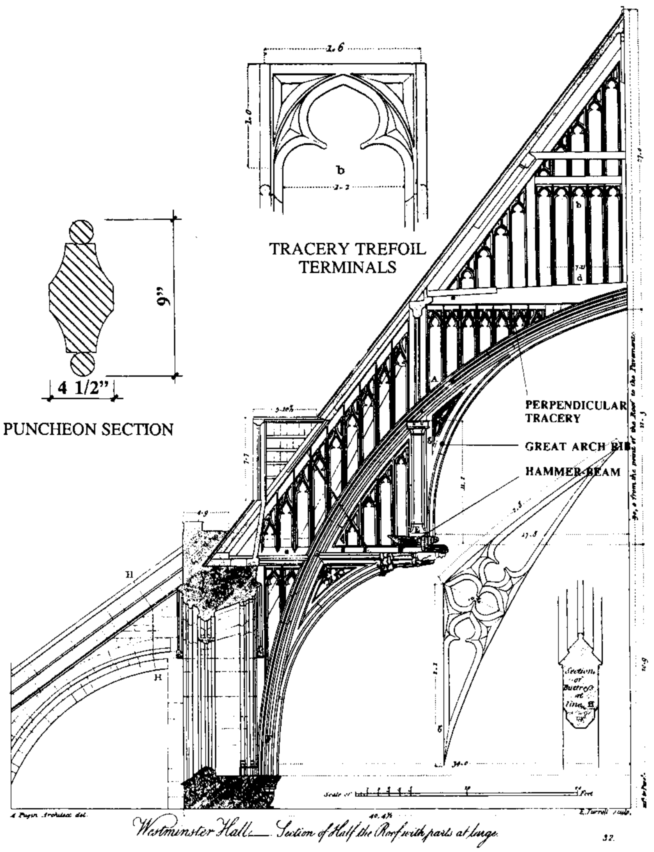


FIGURE 2: A. C. Pugin, truss and tracery details, Westminster Hall roof, from *Specimens of Gothic Architecture* (1895); section of tracery puncheon added by authors.

THE FINITE ELEMENT METHOD OF STRUCTURAL ANALYSIS

The creation of the analytical technique now known today as the finite element method (FEM) spanned a century to combine the computational power of the digital computer with developments in continuum mechanics.[11] FEM breaks down any structural system or continuum into submembers (such as individual truss members or discrete segments of a continuous vault). Computer algorithms then utilize the spatial information locating these discrete elements and the mechanical properties of their materials to assemble a comprehensive model so that an overall structural evaluation can be made (see Appendix: On the finite element method of analysis).

In the field of structural engineering, finite element analysis has rapidly become the standard: it has also been used to study such diverse phenomena as the structure of marine shelled organisms, stresses in bio-engineered human in-plants, the air flow around airplane wings and the forces in a suspension bridge.[12] FEA supersedes many of the former methods historians may be familiar with (including plastic modeling).[13] It is far more precise, cheaper and provides quicker results. It is also more flexible: FEA can be used to investigate entire buildings constructed of any material of known properties (stone, concrete, unit masonry, steel, iron, or wood) or explore the performance of a building's subassemblies (domes, buttresses, or trusses). Finally, because of FEA's computational power, the number of simplifying assumptions necessary for analysis is greatly reduced, leading to more accurate results. These strengths, which have caused FEA to revolutionize engineering practice, are equally beneficial to historic structures research, and as a result FEA is beginning to be used in historical applications.[14]

Perhaps the strongest point for historians without formal training in engineering is that FEA output is graphic and intuitive. Once the structural model is coded and run on a computer the user can essentially view the internal structural workings of the building firsthand, including diagrams of its deflected shape under loading, the axial forces (running parallel with its members), bending moments, and shear forces (running perpendicular to the axes of members). A series of "what if" games can then be played with the working computer model. Changes in support or loading assumptions are easily made and the results quickly obtained. For example, the numerous alterations made to a Gothic cathedral over the centuries can be explored (do they reflect formal or structural concerns?). Herein lies the potential of FEA to transform our understanding of a historic building from one which is focused solely on formal, stylistic, or contextual issues to one in which the real structural problems addressed by the builders and the designers of the time are given their appropriate weight.

BUILDING THE FINITE ELEMENT MODEL

Our method of study was to build a finite element model of the Westminster Hall truss in order to explore its performance and the possible structural contribution of its tracery. This process was interactive and included

- building the basic finite element model and using it to help establish informed assumptions about its construction, loading, and support
- analyzing the structure with and without tracery to isolate its structural contribution
- interpreting the historical significance of our findings.

The basic FEA model was built in two–dimensional Cartesian space (x and y coordinates), in this case defining the geometry of the truss and the location of joining timbers at "nodes." The truss members were represented as model "frame elements," given their specific dimensions and material properties in the original, undecayed truss, and attached to these nodes. With this model in place we were able to explore the behavior of the truss with and without tracery and under varied loading and support assumptions.

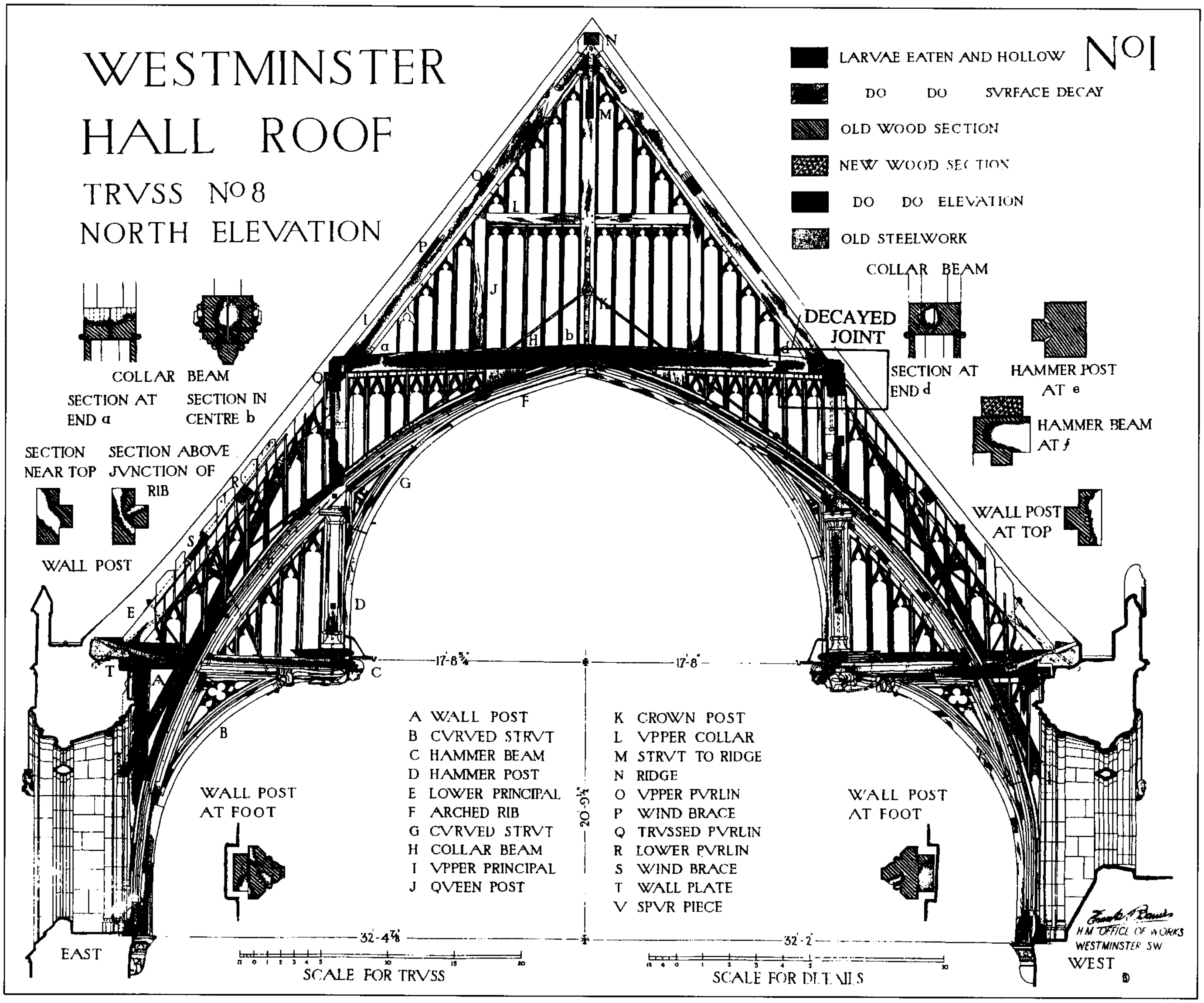


FIGURE 3: Sir Frank Baines, truss elevation and detail of collar beam, Westminster Hall roof, from *Report on . . . Westminster Hall* (1914).

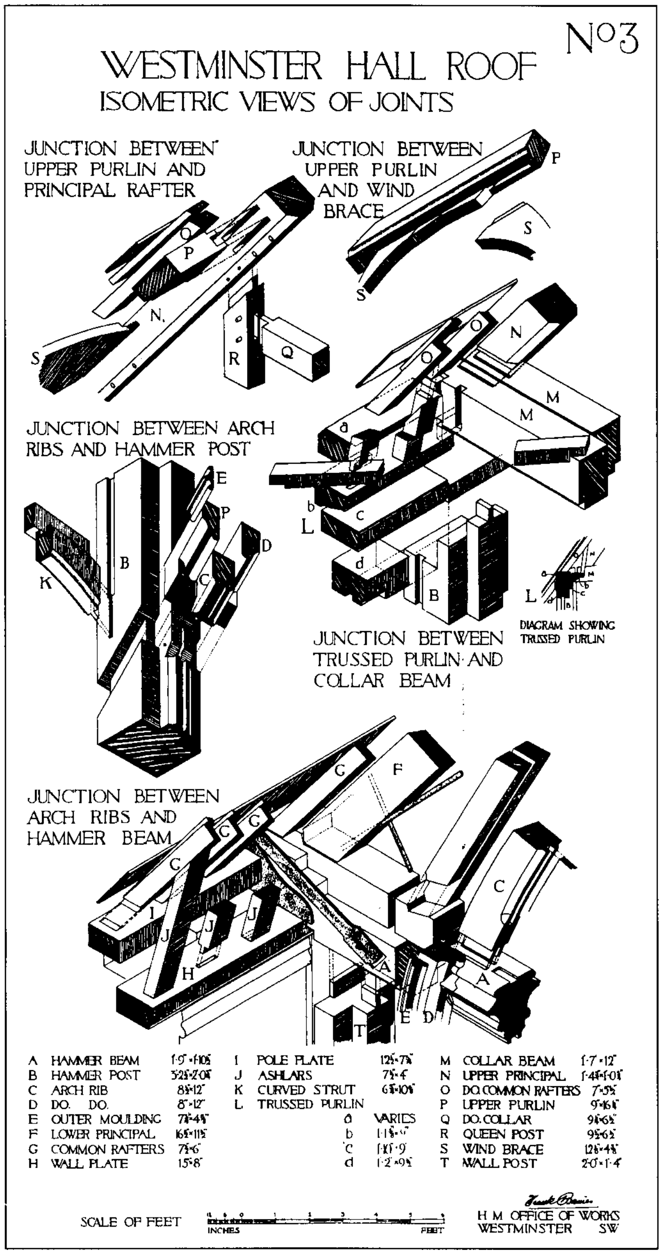


FIGURE 4: Sir Frank Baines, isometric views of joints, Westminster Hall roof, from *Report on . . . Westminster Hall* (1914).

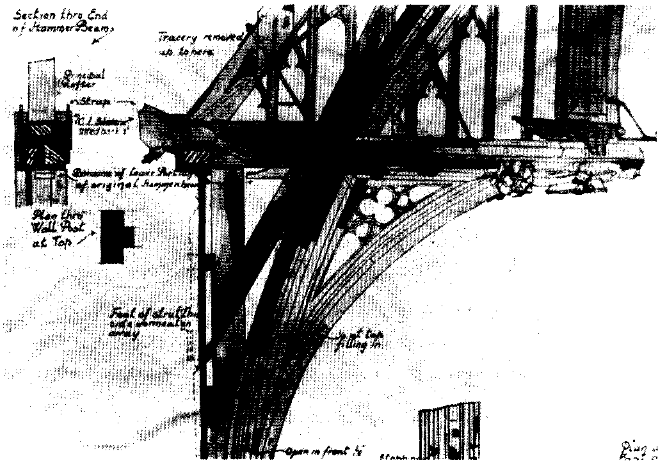


FIGURE 5: Sir Frank Baines, section and elevation of truss at wall head, Westminster Hall roof. Schedule no. 27 made in preparation for truss reinforcements. Note the air gap between the masonry wall and wall post with blocking providing lateral support at its midpoint.

Baines's *Report* and his recently recovered "schedules" (drawings executed in preparation for his restoration of the roof) formed the basis of our model of Herland's 1395–96 truss [*Figures 3–5*].[15] These resources were supplemented by drawings of the truss and its members in works by A. C. Pugin and E. J. Willson, Friedrich Ostendorf, H. Cescinsky, and E. R. Gribble.[16] The large truss members were typically mortised, tenoned, and pegged together, forming stiff joints capable of resisting substantial forces. These connections were modeled as fixed, released, constrained, or partially released, according to the best mathematical representation for the physical condition. For example, the composite great arch is pegged together at many points. This member was modeled as three separate truss-frame members constrained to move together at those pegged points. The connection where the outer laminae of the great arch pass through mortises in the hammer post was modeled as a rigid moment connection, one capable of transmitting rotational forces to all adjoining elements (see Figure 4, detail entitled "Junction between Arch Ribs and Hammer Beam"). The remaining joints, with the exception of the truss connection to the masonry walls, were modeled with three degrees of freedom (free to rotate about the orthogonal z axis and translate in the x–y plane of the truss).

The usefulness of any structural model, with the finite element model no exception, is dependent on accurate assumptions concerning its support and loading. Engineering models are abstractions of the structures they represent; and while the determination of precise stress levels in members may not be the main issue in the analysis of a historic building, the basic assumptions must be grounded in the reality of the structure. But what is this reality?

First is the issue of support. A lively debate between engineers over realistic support assumptions for the truss has effectively been put to rest by the recovery in 1990 of Baines's schedules.[17] During the 1395–96 building campaign the Norman walls were raised and heraldic corbel stones installed approximately 20 feet below the new wall head, providing seating at the base of the truss's wall posts. While each truss appears to bear both on these corbel stones and at the wall head, many support alternatives can be and have been argued.[18] The contested issue among investigators has been where the walls resist the outward thrust of the truss. Baines's schedules provide strong evidence that (contrary to Heyman's hypotheses) no lateral bearing can occur where the truss apparently abuts the inside corner of the wall head. Schedule 27 [*Figure 5*] shows that the upper half of the wall post was set several inches away from the masonry wall and that as a result the notch between the hammerbeam and wall post could never make contact with the wall head.[19] Further, lateral support of the wall post is shown to be provided at its base and by blocking behind its midpoint, at the approximate location of the springing of the great arch. But what of vertical load bearing? In the absence of lateral support at the wall head our computer models indicated little difference in the structural performance of the truss whether or not the hammerbeam is assumed to bear the truss and roof weight on the top of the wall. Hence, we modeled the frame of the truss with support in only two locations: horizontal and vertical support at the wall post's base (the corbel stone) and horizontal support alone at its approximate midpoint.[20] A thorough and invasive site investigation of these conditions or more convincing archival evidence could have resolved the issue of support. Lacking these, FEA was used to test the feasibility of various hypotheses. In this sense it can complement traditional forms of historical evidence.

Early investigators' assumptions about loading were varied and largely arbitrary. In order to simplify his analysis Pippard adopted a pattern of eight point loads for "approximate loadings" [*Figure 6*]. Heyman and Mainstone progressively abstracted Pippard's loading, and Mark, seeking "general behavior," assumed three equal point loads of 10 metric tons applied at the attachment points of the ridge member and the main purlins.[21] As illustrated by Viollet-le-Duc in his *Dictionnaire raisonné de l'architecture française* [*Figure* 7], the roof framing consists of many different–sized purlins braced in various fashions, so that overall, they carry more or less of the roof load and apply forces to the truss according to their stiffness (arrows in figure 7 indicate loading points on the truss). This weight distribution does not result in any regular pattern, as suggested previously. Recognizing that structures are sensitive to loading we performed a separate three-dimensional finite element analysis of the roof to determine a more accurate load pattern with which to model the truss [*Figure 8*].[22] The results of our roof modeling yielded a more complex pattern of self-weight and point loads on the truss [*Figure 9*] than that assumed by previous authors. The trussed purlins were found to carry 72 percent of the roof's vertical loads to the hammer post and hammerbeam. The ridge and the upper and lower purlins carry relatively minor forces to their attachment points on the truss.

RESULTS

Using the loading pattern and support conditions described above we conducted several computer simulations, with and without the presence of tracery. Our studies reveal several characteristics of the behavior of the Westminster Hall trusses and the function of the tracery.

First, the deflected shape diagram indicates what would be expected: the truss, regardless of the presence or absence of tracery, sags under loading from the roof and purlins, exerting an outward thrust on the walls [*Figure 10*]. The FEA graphic output exaggerates the deflected shape to emphasize the direction of movement in the truss and the bowing of its members. These images of bending deformation are the contemporary equivalent of the physical scale models Herland probably used to test the viability of his designs.[23] The only visual description of the flow of forces he could have seen when he pushed on or hung weights from these models would have been those leading to the same types of deflections shown in the FEA image.

Modern engineers associate trusses and truss behavior with axial forces only. When the Westminster Hall truss is examined purely from the point of axial force flow the tracery exhibits little structural contribution [*Figure 11*]. The relative magnitude of forces running parallel with the truss members is indicated in the computer image by thicker bands in those areas of greatest

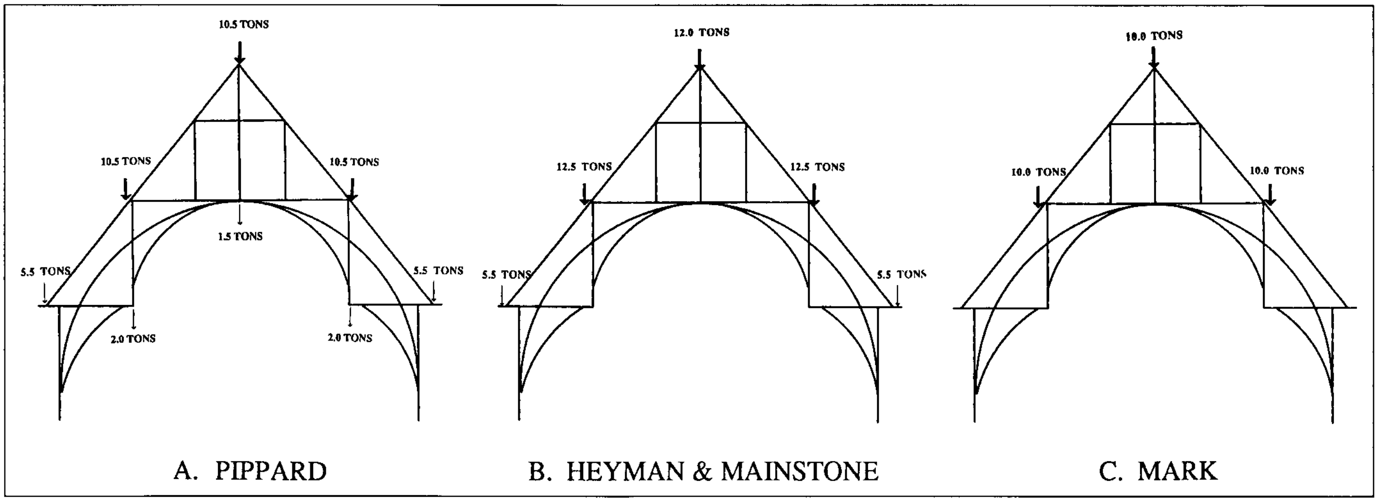


FIGURE 6: Westminster Hall roof, truss loading patterns proposed by previous investigators: (A) Eight–point loading, after A. J. Sutton Pippard and W. H. Glanville (1926); (B) Jacques Heyman and Rowland Mainstone (1967) and Rowland Mainstone (1967); (C) Robert Mark and Huang (1987).

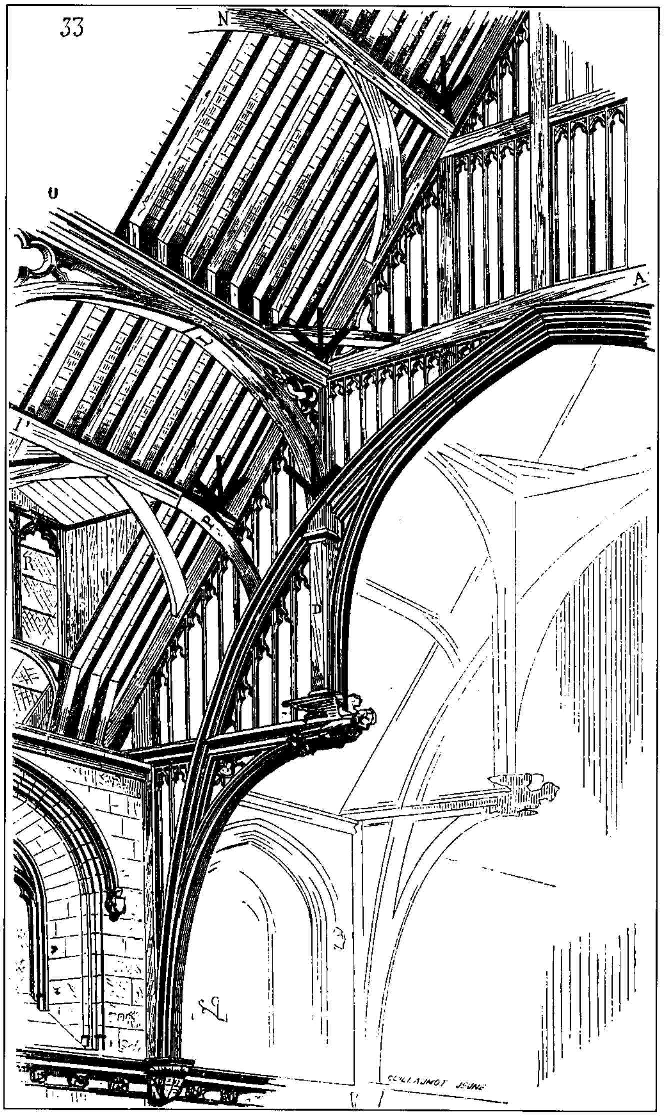
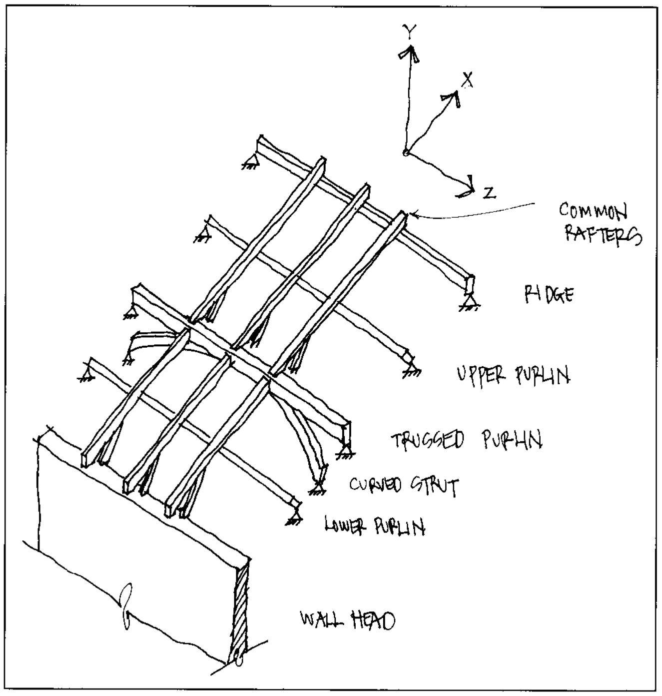


FIGURE 7: E. E. Viollet-le-Duc, detail of framing, Westminster Hall roof, *Dictionnaire raisonné* (1854-1868). Arrows, added by the authors, indicate points of loading from the roof.

axial force. Compression is displayed solid and tension hatched.[24] As Mainstone predicted and as Mark's experimental findings later corroborated, axial or compressive forces in the main members run primarily through the extremely large and stiff great arches to the corbel stones. The hammerbeam, acting like a broken collar tie in tension, resists some of the roof's outward thrust. The presence of tracery has negligible effect on the overall pattern of axial force distribution. The puncheons all carry axial compression (the only type of force they are detailed to carry), slightly reducing the axial force in the queen posts and the hammer post.



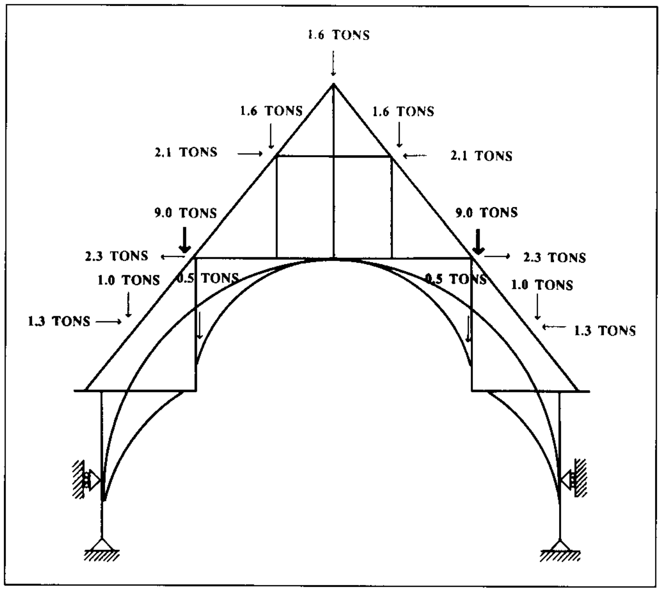


FIGURE 8: Westminster Hall roof, framing schematic. A three-dimensional finite element model was built based on this schematic to determine the truss load pattern (see Fig. 9). Note the five points of connection where roof members apply point loads to the truss. FIGURE 9: Westminster Hall, roof, support and loading diagram for trusses per Baines's Schedule no. 27 and authors' 3-D finite element model of the roof structure.

When bending is considered, however, a different understanding of the tracery emerges. A member, bowed under external forces, experiences internal tensile and compressive stresses in its outermost fibers. The greater the bowing of the member, the greater these stresses. Figure 12 depicts bending moments in the truss with (left) and without (right) tracery. The FEA software used in this study depicts areas of greater bending with more hatching, plotting bending on the concave or compressive side of the bowed member. Many of the truss's members experience bending. These are not insignificant secondary forces but produce maximum stress levels equal to four times those of axial. The tracery absorbs and redistributes loads from the surrounding principal members. Consider again Herland's likely design process. Percy J. Waldram noted:

> When Hugh Herland . . . designed the Westminster Hall roof he did not look up textbooks on structural mechanics. . . . His textbooks and stress diagrams were his innumerable models, which as we know occupied so much space that rooms in the King's palace had to be reserved for them.[25]

Herland may very well have employed the tracery as a result of bending deformation observed in his models. While this speculation cannot be proven, the decision to use the tracery resulted in reduction of bending moments in all the truss's larger members, in some cases by more than 50 percent.

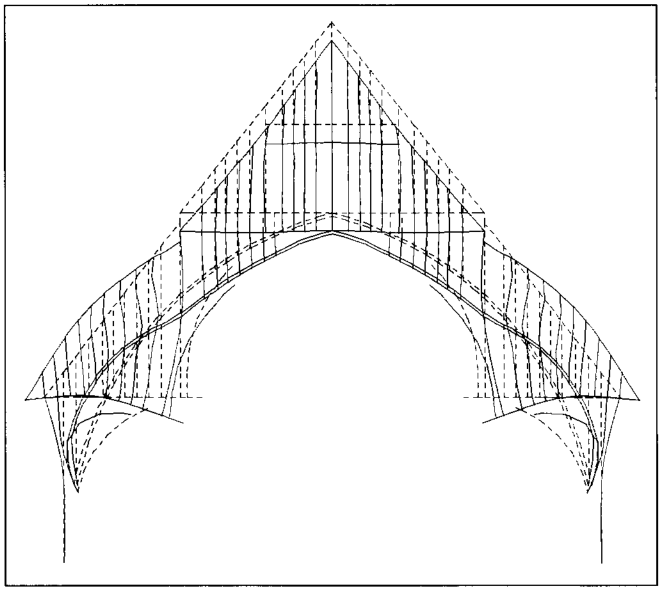
After we had established an overall understanding of the structure's behavior and the contribution of its parts a second stage of analysis was necessary: taking the detailed numerical output and performing checks on individual members to insure that they are capable of assuming the loads the FEA model predicts. At Westminster Hall this process indicated that Herland's original structure was overbuilt many times a sufficient factor of safety. Several structural elements, including some of the large timbers and the tracery itself, could have been reduced in size or eliminated altogether without causing the failure of the roof system.[26] In modern engineering terms, the Westminster Hall truss design was inefficient, employing more material than was required to support the roof loads.

Herland's truss, however, was also a redundant structure, one which over time apparently relied on the tracery to prevent its total collapse. In 1913 Baines found severe degradation of the trusses caused by wood rot. In some trusses, critical joints were so rotted that principal structural members no longer carried any load.[27] Baines noted,

> Should the structure have been one unable to adjust itself to the altered conditions due to defects in the roof, it must have undoubtedly fallen. It is merely by what might be termed the interaction and redundancy of the various members . . . that some of the trusses remain in position. . . . The result is that the collar beam has transmitted its load down the puncheons of the tracery immediately beneath it onto the great curved rib.[28]

The longevity of Herland's roof results in part from the oversizing of its members and in part from the redundancy of its design. Even today redundancy is considered an important feature of all structures, but it is essential in those of long spans or innovative design.

The tracery's capacity to carry huge axial loads, its function in reducing bending, and its role as a secondary structural system ensuring the viability of the roof led us to speculate whether Herland could have considered the tracery a means of strengthening his roof at Westminster. Perpendicular tracery, although new at that time, had been used forty years previously in conjunction with the double arch structure at St. George's Hall in Windsor [*Figure 13*]. Given the stylistic similarities between Windsor and Westminster and the likelihood that Herland knew Windsor intimately, it has been called "probably the single most important source for the Westminster Hall design."[29] Windsor appears to have consisted only of slight upper and lower arches with heavy tracery infill, highly suggestive that its tracery was employed with a structural intent. Unfortunately, only poor documentation of this roof survives and analysis of the structure is impossible.[30] As an independent exploration, therefore, we built and analyzed a finite element model of the Westminster truss reduced to the configuration of Windsor [*Figure 14*]. All so-called principal members absent from the Windsor design (the hammerbeam and post, the brackets, and all members above the collar beam) were removed, producing a model with an arch rib, collar beam, rafters, and Perpendicular tracery, strikingly similar to the



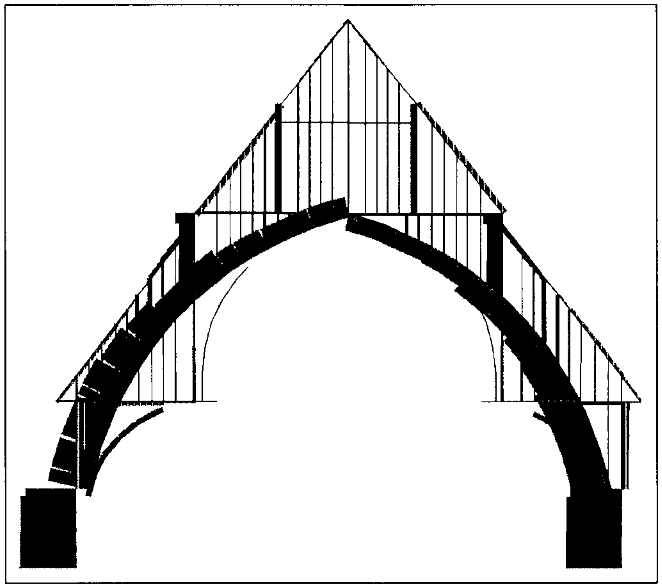


FIGURE 10: Westminster Hall roof, finite element model showing deflected shape in traceried truss caused by vertical loading. FIGURE 11: Westminster Hall roof, finite element model showing axial forces in traceried truss.

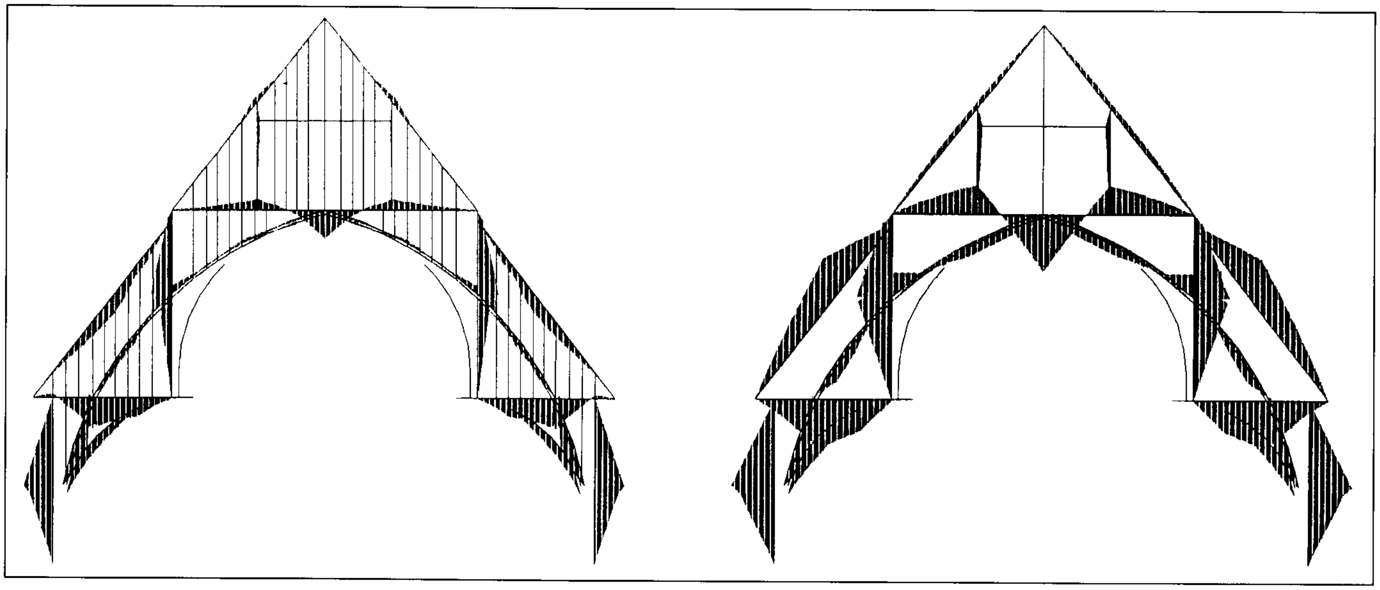


FIGURE 12: Westminster Hall roof, finite element models showing comparison of bending moments in truss with tracery (left) and without tracery (right).

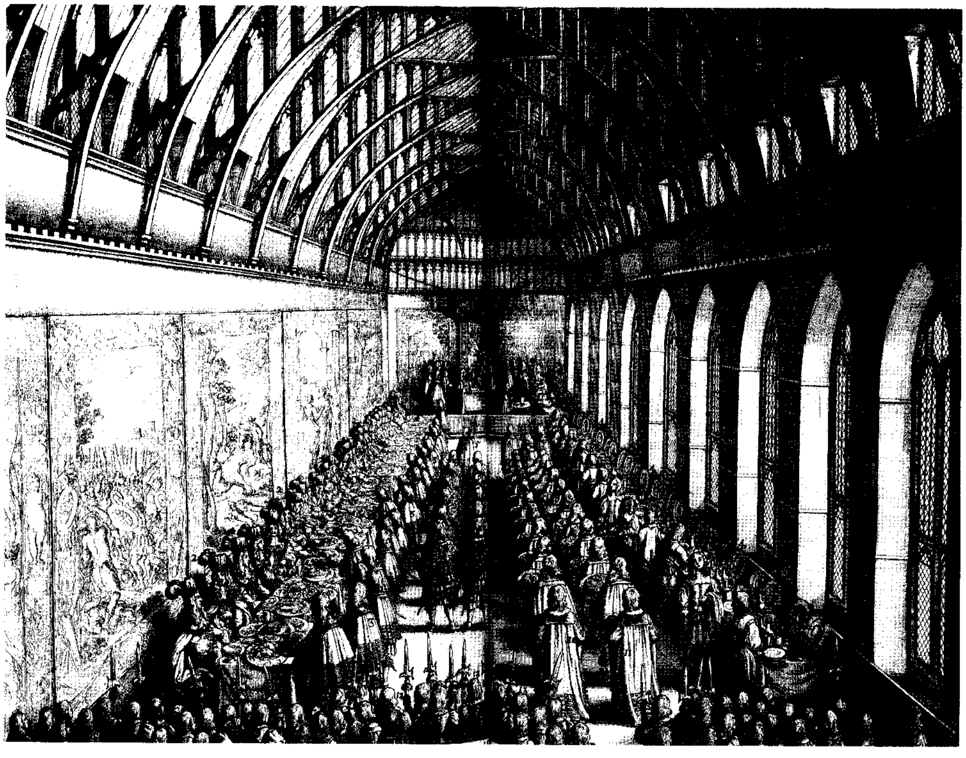
Windsor truss. This model proved structurally viable and depended upon the tracery for its stability. Although Herland could not have seen the flow and magnitude of these forces, his familiarity with the use of tracery at Windsor and his own structural investigations for the Westminster Hall roof design may well have led him to an understanding of the structural contribution of the tracery.

Conclusions

At first impression Herland's truss is aesthetically delicate and structurally robust. It is a harmonious structure of exquisite detail, and structural analysis confirms overall appearances: the prominent and extremely stiff arch is responsible for discharging most of the roof loads to the corbel stones and heavy masonry walls below. What is less evident, however, is that many of its decorative aspects are also load bearing and that the demands placed on them have changed dramatically over the life of the structure. What structural role Herland intended for the tracery we may never know, but his decision as master builder to include it in his truss design was consistent with sound engineering practices which have stood the test of time.

The traditional training of architectural historians and of

FIGURE 13: Windsor Castle, former St. George's Hall (c. 1362–65), engraving by Wenceslas Hollar, 1672.



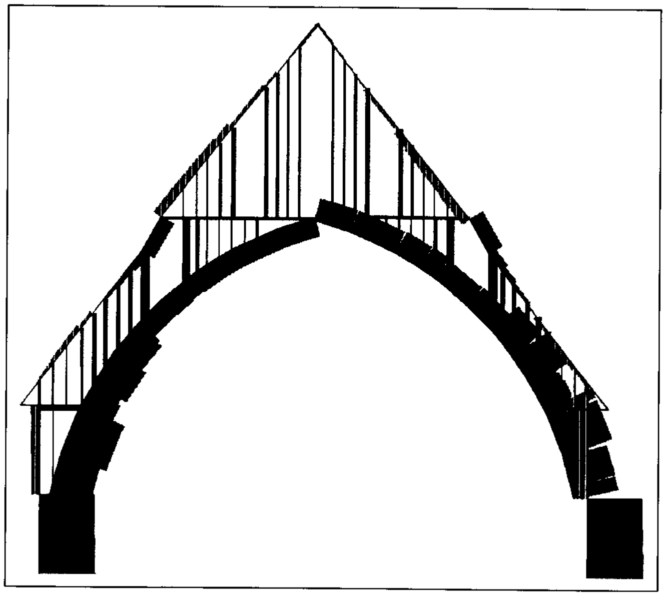


FIGURE 14: Westminster Hall roof, hypothetical finite element model of the truss reduced to the configuration of St. George's Hall at Windsor by the removal of key members. Axial forces shown.

engineers may be ill-adapted to research that attempts to illuminate the full range of issues faced by early builders. The case study of Westminster Hall illustrates the capacity of FEA to bridge historical and technical explorations. More than simply offering analytical power—essential in establishing the evolution of structural innovation—FEA in concert with historical research provides insight into possible relationships between structural and formal developments.

Appendix

On the finite element method of analysis

The working principle of the finite element method (FEM) is the evaluation of the stiffness properties of structural elements based on assumed sets of displacement interpolation functions (mathematical functions which relate the displacement of an element to its corresponding internal forces). The computer analysis is based on a model (an abstract, mathematical construction which can be represented schematically) defined in Cartesian space and constructed to represent the building structure or its specific parts. Its principal components are nodes, elements, applied external forces, and specified nodal restraints [*Figure 15*]:

- "Nodes" describe the actual geometry of a structure and are rigid bodies that occupy points in Cartesian space. They have six potential degrees of freedom: three translational degrees of freedom parallel to the Cartesian axes, and three rotational degrees of freedom about the Cartesian axes.
- "Elements" are deformable bodies that idealize real structural members, and can be of different types (truss elements, frame elements, shell elements, and solid elements). Elements have defined stiffness properties which are derived directly from the real geometrical properties (moment of inertia, cross–sectional area, length) and real material properties (modulus of elasticity, shear modulus, coefficient of thermal expansion) of the members they represent.
- "Applied external forces" may be specified to act either on individual nodes (as with concentrated or point loads) or individual elements (such as distributed loads). They model real loads experienced by the structure such as

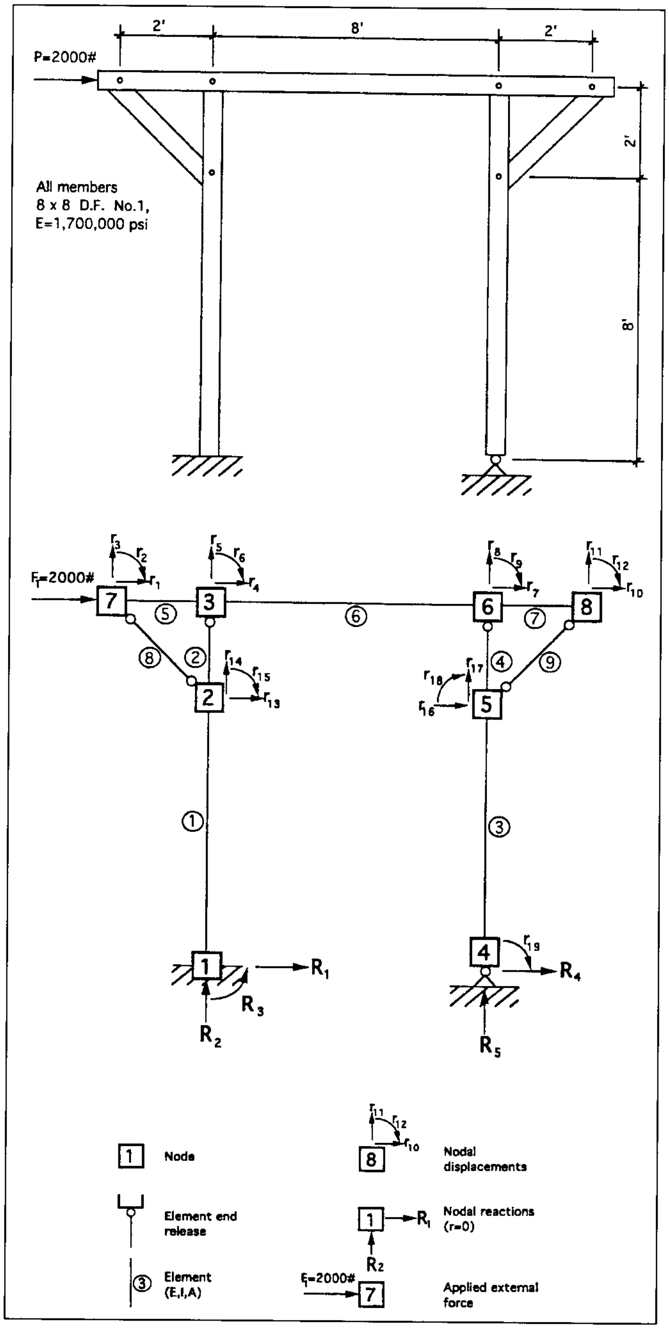
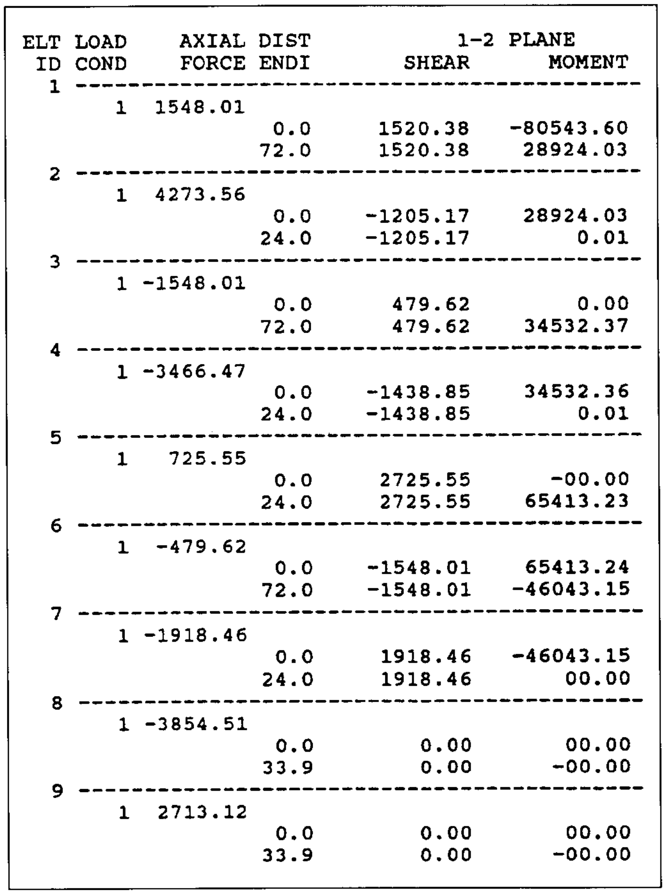


FIGURE 15: Finite element model of a simple frame used to build a computer input file for analysis: the schematic drawing, the idealized finite element model, and model components.



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ELT LOAD     AXIAL DIST              1-2 PLANE
 ID COND     FORCE ENDI           SHEAR          MOMENT
1 ---------------------------------------------------
      1  1548.01
                   0.0         1520.38     -80543.60
                  72.0         1520.38      28924.03
2 ---------------------------------------------------
      1  4273.56
                   0.0        -1205.17      28924.03
                  24.0        -1205.17          0.01
3 ---------------------------------------------------
      1 -1548.01
                   0.0          479.62          0.00
                  72.0          479.62      34532.37
4 ---------------------------------------------------
      1 -3466.47
                   0.0        -1438.85      34532.36
                  24.0        -1438.85          0.01
5 ---------------------------------------------------
      1   725.55
                   0.0         2725.55        -00.00
                  24.0         2725.55      65413.23
6 ---------------------------------------------------
      1  -479.62
                   0.0        -1548.01      65413.24
                  72.0        -1548.01     -46043.15
7 ---------------------------------------------------
      1 -1918.46
                   0.0         1918.46     -46043.15
                  24.0         1918.46         00.00
8 ---------------------------------------------------
      1 -3854.51
                   0.0            0.00         00.00
                  33.9            0.00        -00.00
9 ---------------------------------------------------
      1  2713.12
                   0.0            0.00         00.00
                  33.9            0.00        -00.00
```

FIGURE 16: Finite element analysis for the simple frame in figure 15: numerical output of element forces.

dead and live loads, the structure's self-weight, wind, seismic loads and temperature effects.

- "Specified nodal restraints" are imposed by the model builder to reflect the capacity of the real building's components to displace or rotate. They represent the support conditions of the real structure, typically defined as fixed ends, pins, rollers, and springs.[31]

Prior to constructing the analysis model research must be conducted to determine the specifics of individual connections, support, and loading conditions. The model is then written as an input file in an appropriate format for the software being used and the components listed above are assembled, connecting appropriate elements to nodes. The real connections of the structure are modeled by specifying "end releases" between elements and nodes which determine what types of forces (axial, shear, moment, torsion) a given element has the capacity to transfer to the node. The basic model is complete after the restraints and all external actions (applied loads, support conditions, and any temperature loads) are defined.

The analysis is run on the computer. Once the run is complete, the computer gives back solutions to three response quantities: nodal displacements, support reactions, and element end forces or stresses. These results are available in numerical form [*Figure 16*], but most software packages now supply at least some of the output graphically, such as the schematic representations of the Westminster truss's deflected shape, axial, and moment force diagrams presented in this paper [see Figures 10–12, 14]. Finally, element checks performed following the computer analysis verify whether the structure's members and actual joints are capable of withstanding the determined forces and stresses. This procedure is standard in engineering design practice and serves as an important check of structural hypotheses.

One caveat must be stated: The results of the computer analysis are limited by the quality of the assumptions made in formulating the model. Regardless of the accuracy and sophistication of the computer application used, FEA results should be independently confirmed by making a series of spot checks (statics calculations) to insure that the mathematical abstraction resembles the real structure.[32]

Notes

This study is an outgrowth of an experimental course combining architectural history and engineering taught in 1991 in the Department of Architecture at the University of California at Berkeley by Professors R. Gary Black (architect and engineer) and Stephen O. Tobriner (architectural historian), with graduate student instructor Stephen Duff (Ph.D. candidate in structures). The concept for this course was proposed by Duff. Thirty graduate students used FEA coupled with historical research to study the structural and spatial attributes of a building or bridge. Among structures investigated were the cathedrals of Amiens and Chartres; the Ralph Symons Trinity College Hall roof; the stave church at Borgund in Norway; Henri Labrouste's Bibliothèque National and Biblioteque Sainte-Geneviève in Paris; several bridges, including Robert Maillart's Schwanbach bridge in Switzerland; nineteenth-century steel-frame high-rises in San Francisco; the Toshintei teahouse on the grounds of the Minase Shrine, Osakai and the picture hall of Sanzo-in at the Yakushiji complex, Heijo. E. Toby Morris (architect and master's candidate in architecture) investigated Hugh Herland's trusses at Westminster Hall. Partial funding for the course came from the University of California, Berkeley's committee on teaching, to which the authors express their thanks. The results of this study were first presented by Black and Morris at the session entitled Buildings as Artifacts, Society of Architectural Historians annual meeting in Charleston, South Carolina, April 1993.

[1] See the writings of Robert Mark in the United States, and of Jacques Heyman and Rowland Mainstone in Great Britain, among others.

[2] Hand calculations can easily be performed for statically determinate structures (those in which there are no more than three unknown forces which can be found using the basic equations of statics). In such cases the analysis is very accurate. Before the development of computer methods, engineers simplified complex, statically indeterminate structures to simpler indeterminate ones, or to ones made up of statically determinate components. The accuracy of the results depended upon the degree to which the simplification was representative of the real structure.

[3] For a summary of engineering and historical studies of the Westminster roof structure over the last century and a half see Lynn T. Courtenay and Robert Mark, "The Westminster Hall Roof: A Historiographic and Structural Study," *JSAH* 46 (1987): 374-93.

[4] Sir Frank Baines, *Report to the First Commissioner of H. M. Works, Etc., on the Condition of the Roof Timbers of Westminster Hall, with Suggestions for Maintaining the Stability of the Roof*, Commissioned Document, House of Commons, 7436 (London, 1914), hereafter cited as Baines, *Report*.

[5] Baines attempted a "stress diagram" for the framing based on graphical statics but disclaimed his results, finding that the theoretical stresses corresponded in no way with the observed traces of actual stresses in the structure.

[6] A. J. Sutton Pippard and W. H. Glanville, "Primary Stresses in Timber Roofs, with Special Reference to Curved Bracing Members," *Building Research Technical Paper 2* (London, 1926): 15–32. Also later published in A. J. Sutton Pippard, *Strain Energy Methods of Stress Analysis* (London, 1928).

[7] See Jacques Heyman, "Westminster Hall Roof," *Proceedings of the Institution of Civil Engineers* 37 (1967): 137–162, and Rowland Mainstone, "Discussion of 'Westminster Hall Roof' by Jacques Heyman," *Proceedings of the Institution of Civil Engineers* 38 (1967): 788–92.

[8] See Courtenay and Mark, "The Westminster Hall Roof," 374–93.

[9] See comments by Mainstone, Heyman, and Mark in "Letters," *JSAH* 47 (1988): 321–24.

[10] Engineers Pippard, Heyman, Mainstone, and Mark each discounted the structural role of the tracery, excluding it from their analyses. According to Courtenay the Perpendicular tracery in English fourteenth-century timber roofs has primarily formal (not functional) derivations, coming from the shapes and details masons used to articulate their window openings. See Lynn T. Courteney, "The Westminster Hall Roof and Its 14th Century Sources," *JSAH* 43 (1984): 309.

[11] In 1909, Ritz presented an approximate method to solving problems in continuum mechanics (the science of determining the exact force flow through a continuous solid body), the most significant contribution to the development of FEM. However, while effective, Ritz's method proved impractical before the advent of the digital computer (1950s). R. W. Clough adapted Ritz's method to the computer to solve two–and three–dimensional problems in continuum mechanics and in 1960 named his innovation the "finite element method." In 1961, Edward L. Wilson wrote the first finite element program. The authors used a recent version of this program developed by Wilson and Ashraf Habibullah (*SAP 90 TM, 1991*) to study Westminster Hall. For a general text on the history, theory, and application of FEA (including extensive references) see R. D. Cook, *Concepts and Applications of Finite Element Analysis* (New York, 1989).

[12] Y. Song, R. G. Black and J. H. Lipps, "Morphological Optimization in the Largest Living Foraminiferia: Implications from Finite Element Analysis," *Paleobiology*, 20 (1994): 14–26; M. J. Turner and R. W. Clough, "Stiffness and Deflection Analysis of Complex Structures," *Journal of Aeroscience* 23 (1956): 805–23.

[13] Robert Mark, "Dual Light Source for a Large Field Diffused Light Polariscope," *Review of the Scientific Instruments* 35 (1964): 521–22; Robert Mark, *Experiments in Gothic Structure* (Cambridge, 1982), 18–33.

[14] S. Kato et al., "Finite-Element Modeling of the First and Second Domes of Hagia Sophia"; Robert Mark, A. Cakmak, and M. Erdik, "Preliminary Report on an Integrated Study of the Structure of Hagia Sophia"; and G. Penelis et al., "The Rotunda of Thessaloniki: Seismic Behavior of Roman and Byzantine Structures," in *Hagia Sophia from the Age of Justinian to the Present*, Robert Mark and A. S. Camak, eds. (Cambridge, England, and New York, 1992).

[15] Besides his published report, Baines produced a collection of so-called schedules: thirty-six numbered and annotated drawings of the trusses and details, only a few of which were included in his report to Parliament. See Lynn T. Courtenay, "The Westminster Hall Roof: A New Archaeological Source," *Journal of the British Archaeological Association* 143 (1990): 95–111.

[16] See A. C. Pugin and E. J. Willson, *Specimens of Gothic Architecture, selected from various Ancient Edifices in England*, vol. I (Edinburgh, 1895); H. Cescinsky and E. R. Gribble, "Westminster Hall and Its Roof," *Burlington Magazine* 40 (1922): 76–84; and Friedrich Ostendorf, *Die Geschichte des Dachwerks* (Leipzig and Berlin, 1908). From the elevations and detailed sectional drawings in these works the complicated history of repairs and alterations to the original structure could be followed and a model of Herland's truss constructed.

[17] For debate on appropriate support modeling assumptions for the trusses see comments by Mainstone, Heyman, and Mark in "Letters" (see n. 9): 321–24.

[18] Heyman's 1967 theoretical explanation of force paths in the structure (one which found the primary rafters to carry the bulk of the roof's loads) depended upon an assumption that the trusses were restrained from outward displacements by their rigid connection to the wall head (Heyman, "Westminster Hall Roof" [see n. 7], 137–62). Mark tested this assumption and other restraint combinations on his physical scale model in order to establish plausible support to the truss (Courtenay and Mark, "The Westminster Hall Roof," 390).

[19] The only other place lateral resistance could have been applied to the hammerbeam is at its outermost end. Drawings in Baines, *Report*; Cescinsky and Gribble, "Westminster Hall"; Ostendorf, *Geschichte des Dachwerks*; and Pugin and Willson, *Specimens*, all fail to document this condition. A drawing by E. E. Viollet-le-Duc (*Dictionnaire raisonné de l'architecture française du XIe au XVIe siècle*, 10 vols. [Paris, 1854–68], 3, under "Charpente," pl. 32) suggests there may be no masonry whatsoever behind the hammerbeam. Even if masonry were so located it would be little more than a few inches thick, easily displaced by lateral forces applied by the truss.

[20] At this point conclusive evidence of the precise location of the original lateral bearing for the midpoint of the wall post is lacking. Courtenay hypothesizes that originally spurs were located approximately 11 feet above the base of the wall post at the intersection with the arch rib, providing lateral support at this juncture, but because of rot their existence and extent are unclear in Baines's schedules. See Courtenay, "The Westminster Hall Roof: A New Archaeological Source," 105.

[21] Courtenay and Mark, "The Westminster Hall Roof," 385. Engineer R. J. Ashby was the first to point out the inaccuracy of Heyman's arbitrarily assigned loads, in "Discussion of 'Westminster Hall Roof' by Jacques Heyman," *Proceedings of the Institution of Civil Engineers* 38 (1967): 785–6.

[22] Our model consisted of upper and lower common rafters with their attendant ashlar pieces, the ridge beam, and the three purlins (upper, trussed, and lower). Distributed force patterns, derived from the dead loads of the original lead (later slate) roofing and self-weight of the rafters, were applied to the individual common rafters. The common rafters transmit their loads to the various purlins, ridge beam, and wall head, each given their appropriate member properties relating to the stiffness of the member. The end reactions of these elements became the point loads that were ultimately used in the investigation of the truss.

[23] P. J. Waldram, "Science and Architecture: Wren and Hooke," *Journal of the Royal Institute of British Architects* 42 (1935): 558.

[24] The axial forces are shown on the top side of a member in some cases and on the bottom in others. This is an irrelevant consequence of the way the nodes were numbered when constructing the basic computer model.

[25] Waldram, "Science and Architecture," 558.

[26] Maximum axial stresses in the large members of the truss with or without tracery typically run on the order of 100 psi, while bending stresses run between 200 and 400 psi. These values for axial and bending are several times less than the magnitudes required to initiate failure caused by overstressing.

[27] Baines, *Report*, 19. "Occasionally large cavities have developed both in the collar beams and in the feet of the principal rafters which join them, and also in the heads of the hammer-posts. So serious is this that in two instances it would appear that the tracery alone is supporting the collar beam by transferring its load to the great curved rib."

[28] Ibid., 33. A finite element model indicated that this revised structure (with the hammer post removed and tracery alone supporting the loads of the collar beam and upper rafter) is viable, with no overstressing of remaining members.

[29] Courtenay, "Westminster Hall Roof and Its 14th Century Sources," 308.

[30] Only a perspective engraving by W. Hollar, published in Elias Ashmole's *Order of the Garter* (London, 1672) survives.

[31] For a discussion of these general structural terms see F. P. Beer and E. R. Johnston, *Vector Mechanics for Engineers; Statics and Dynamics* (New York, 1972) or any general text on statics.

[32] In presenting their finite element analysis program, Wilson and Habibullah published thirty-three examples of engineering problems (including twisted beams, vibrating plates, domes, and cooling towers) in which the accuracy of their finite element solutions is compared to known classical (exact mathematical) solutions. Wilson and Habibullah, *SAP 80 Structural Analysis Program, A Series of Computer Programs for the Static and Dynamic Finite Element Analysis of Structures, Sample Example and Verification Manual*, Berkeley, 1986.

Illustration Credits

Figure 1. Sir Frank Baines, *Report to the First Commissioner of H. M. Works, Etc., on the Condition of the Roof Timbers of Westminster Hall, with Suggestions for Maintaining the Stability of the Roof*, Commissioned Document, House of Commons, 7436 (London, 1914), pl. 7.

Figure 2. A. C. Pugin and E. J. Willson, *Specimens of Gothic Architecture, selected from various Ancient Edifices in England*, vol. 1 (Edinburgh, 1895), pl. 32.

Figure 3. Baines, *Report*, pl. 1.

Figure 4. Baines, *Report*, pl. 3.

Figure 5. L. T. Courtenay, "The Westminster Hall Roof: A New Archaeological Source," *Journal of the British Archaeological Association* 143 (1990), fig. 5.

Figures 6, 8, 9, 10, 11, 12, 14. Authors.

Figure 7. E. E. Viollet-le-Duc, *Dictionnaire raisonné de l'architecture française du XIe au XVIe siècle*, 10 vols. (Paris, 1854–1868), "Charpente," vol. 3, pl. 33.

Figure 13. Elias Ashmole, *The Institution, Laws & Ceremonies of the Most Noble Order of the Garter* (London, 1672), following p. 592.

Figures 15, 16. R. Gary Black and Stephen Duff, "A Model for Teaching Structures: Finite Element Analysis in Architectural Education," *Journal of Architectural Education*, 48 (September 1994), fig. 4.